WRITE

$$m(t) = \begin{cases} 0, & t < 2 \\ t^2 \xleftarrow{f(t)}, & 2 < t < 5 \\ t^3 + 2t^2 \xleftarrow{f(t)+g(t)}, & 5 < t < 6 \\ t^4 - 2t^3 \xleftarrow{f(t)+g(t)+h(t)}, & t > 6 \end{cases}$$

IN HEAVISIDE NOTATION

$$u(t-2)\,t^2 + u(t-5)(t^3+2t^2-t^2) + u(t-6)(t^4-2t^3-(t^3+2t^2))$$

$$= u(t-2)t^2 + u(t-5)(t^3+t^2) + u(t-6)(t^4-3t^3-2t^2)$$

$$\mathcal{L}\{u(t-a)f(t)\} = \int_0^\infty e^{-st}\,u(t-a)f(t)\,dt$$

$$= \int_a^\infty e^{-st} f(t)\,dt$$

LET $v = t-a$ $\begin{cases} t=\infty \to v=\infty \\ t=a \to v=a-a=0 \end{cases}$

$t = v+a$

$dt = dv$

$$= \int_0^\infty e^{-s(v+a)} f(v+a)\,dv$$

$$= \int_0^\infty e^{-sv} e^{-sa} f(v+a)\,dv$$

$$= e^{-as}\int_0^\infty e^{-sv} f(v+a)\,dv$$

$$= e^{-as}\int_0^\infty e^{-st} f(t+a)\,dt = e^{-as}\mathcal{L}\{f(t+a)\}$$

IF $f(t) = \begin{cases} 0, & t<2 \\ t, & t>2 \end{cases}$, FIND $\mathcal{L}\{f\}$

$= u(t-2)(t-0)$

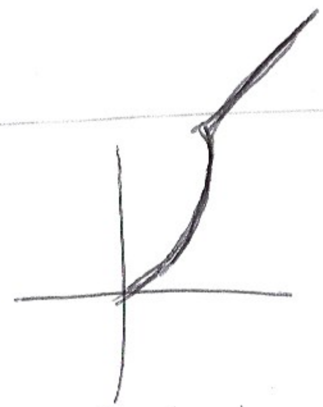
$= u(t-2)t$

$$\mathcal{L}\{u(t-2)\,t\} = e^{-2s}\mathcal{L}\{g(t+2)\} = e^{-2s}\left(\frac{1}{s^2}+\frac{2}{s}\right)$$

$a=2$ $g(t)=t$

$g(t+2)=t+2$

IF $f(t) = \begin{cases} t, & 0<t<1 \\ t^2, & 1<t<3 \\ 3t, & 3<t \end{cases}$



THEN $f(t) = t + \begin{cases} 0, & 0<t<1 \\ t^2-t, & 1<t<3 \\ 3t-t, & t>3 \end{cases}$

$= t + u(t-1)(t^2-t) + u(t-3)(3t-\cancel{t}-(t^2-\cancel{t}))$

$= t + u(t-1)(t^2-t) + u(t-3)(3t-t^2)$

SO (CONT'D)

$$\mathcal{L}\{t + u(t-1)(t^2-t) + u(t-3)(3t-t^2)\}$$

$g(t) = t^2 - t$ $\qquad$ $h(t) = 3t - t^2$

$g(t+1) = (t+1)^2 - (t+1) = t^2 + t$ $\qquad$ $h(t+3) = 3(t+3) - (t+3)^2 = -t^2 - 3t$

$$= \frac{1}{s^2} + e^{-s}\left(\frac{2}{s^3} + \frac{1}{s^2}\right) + e^{-3s}\left(-\frac{2}{s^3} - \frac{3}{s^2}\right)$$

$$\mathcal{L}^{-1}\Big\{\underbrace{e^{-s}}_{e^{-as}}\underbrace{\left(\frac{2}{s^3} + \frac{1}{s^2}\right)}_{\mathcal{L}\{f(t+a)\}}\Big\} = u(t-1)(t^2-t)$$

$a = 1$ $\qquad$ $= \mathcal{L}\{f(t+1)\}$

$$\mathcal{L}^{-1}\left\{\frac{2}{s^3} + \frac{1}{s^2}\right\} = t^2 + t$$

$$f(t+1) = t^2 + t$$

$$f((t-1)+1) = (t-1)^2 + (t-1)$$

$$f(t) = t^2 - t$$

$$\mathcal{L}^{-1}\{e^{-as}F(s)\}$$

IDENTIFY a $\qquad$ $F(s) = \mathcal{L}\{g(t+a)\} =$ ..

$\mathcal{L}\{$ $\qquad$ $g(t+a) = \mathcal{L}^{-1}\{F(s)\}(t)$

$$g((t-a)+a) = \mathcal{L}^{-1}\{F(s)\}(t-a)$$

$$g(t) = \mathcal{L}^{-1}\{F(s)\}(t-a)$$

$$= u(t-a)\,\mathcal{L}^{-1}\{F\}(t-a)$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\left(-\frac{2}{s^3}-\frac{3}{s^2}\right)\right\}$$

$$\downarrow$$

$$u(t-3)\,(-t^2+3t)$$

$$\mathcal{L}^{-1}\left\{-\frac{2}{s^3}-\frac{3}{s^2}\right\}$$

$$= -t^2-3t$$

$$-(t-3)^2-3(t-3)$$

$$-t^2+3t$$

SOLVE $y' + y = \begin{cases} 1; & x < 1 \\ 2; & x > 1 \end{cases}, \quad y(0) = 3$

$$= 1 + u(t-1)(2-1) = 1 + u(t-1)$$

$\leftarrow f(t) = 1$

$f(t+1) = 1 \rightarrow \mathcal{L} = \frac{1}{s}$

$$sY - \cancel{y(0)}^{3} + Y = \frac{1}{s} + e^{-s} \cdot \frac{1}{s}$$

$$\frac{\cancel{(s+1)}Y = 3 + \frac{1}{s} + e^{-s} \cdot \frac{1}{s}}{s+1} = \frac{3}{s+1} + \frac{1}{s(s+1)} + e^{-s} \cdot \frac{1}{s(s+1)}$$

$$Y = \frac{3}{s+1} + \underbrace{\frac{1}{s} - \frac{1}{s+1}}_{} + e^{-s}\underbrace{\left(\frac{1}{s} - \frac{1}{s+1}\right)}_{}$$

$$y = 3e^{-t} + \underbrace{1 - e^{-t}}_{} + u(t-1)(1 - e^{-(t-1)})$$

$$= 2e^{-t} + 1 + u(t-1)(1 - e^{-(t-1)})$$

$$= \begin{cases} 2e^{-t} + 1, & \text{IF } t < 1 \\ 2e^{-t} + 1 + 1 - e^{-(t-1)}, & \text{IF } t > 1 \end{cases}$$

$$= \begin{cases} 2e^{-t} + 1, & \text{IF } t < 1 \\ 2e^{-t} - e^{-(t-1)} + 2, & \text{IF } t > 1 \end{cases}$$

$e^{-t}e$

$(2-e)e^{-t} + 2$

$$\frac{1}{s(s+1)} = \frac{\cancel{A}^{1}}{s} + \frac{\cancel{B}^{-1}}{s+1}$$

$$1 = A(s+1) + Bs$$

$s = 0: \; 1 = A$

$s = -1: \; 1 = -B \rightarrow B = -1$

MANDATORY SANITY CHECK

$s = -2: \; \frac{1}{(-2)(-1)} = \frac{1}{2}$

$\frac{1}{-2} - \frac{1}{-1} = -\frac{1}{2} + 1 = \frac{1}{2}$ ✓